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16. ABSTRACT

This paper involves a problem confronted by the working diatom taxonomist who must make some basic observations and measurements of the diatom cell wall to achieve an accurate identification. The identification of diatoms to the species level is possible by microscopic observation and measurement of the intricate patterns on the cell wall. Since diatoms form the beginning of many aquatic food chains; they are important in many environmental studies and diatom communities can be used to assess the integrity or health of aquatic ecosystems.

A morphological feature used in identification of centric diatoms is the number of striae found in a 10 micron arc along the edge of the diatom. This measurement is made by using a reticule in the microscope eyepiece. For large cells the arc and the 10 micron measuring cord become essentially identical. But for small cells the arc becomes increasingly larger than the 10 micron measuring cord, resulting in a distortion of the measured striae density. The solution of the problem of determining at what cell size a different method of determining the striae density must be used is presented.

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An elementary mathematical problem arising in diatom taxonomy

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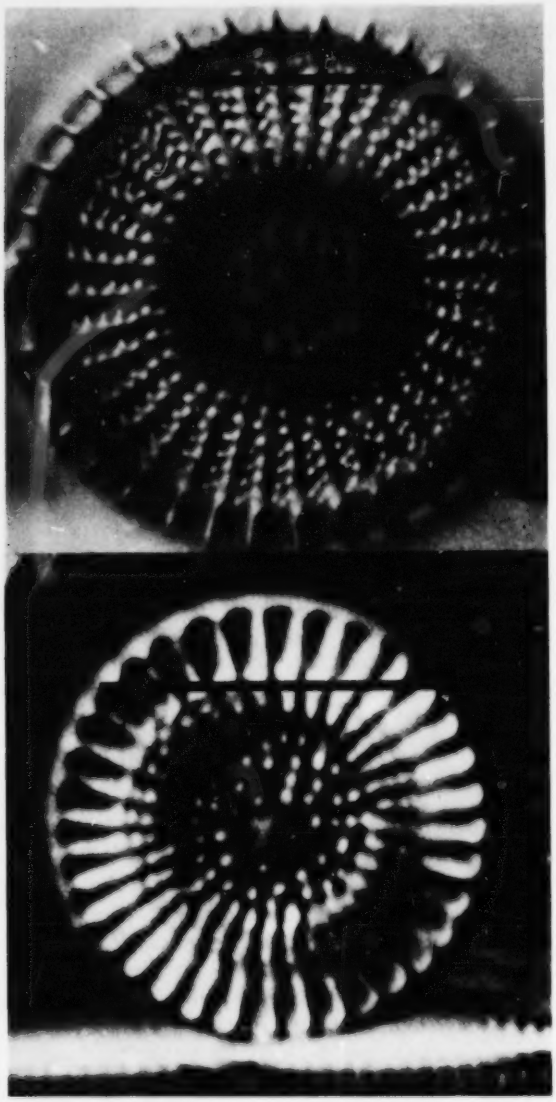
In this paper the authors outline the elementary mathematics involved in a problem that occurs in the taxonomy of micro-organisms.

The purpose of this brief note is to call attention to an elementary mathematical discussion of a problem occurring in the taxonomy of micro-organisms. Elementary trigonometry and analysis of functions are the only mathematics involved, and the problem reduces to that of finding a fixed point of a simple function. This example has been used by the second author in elementary numerical analysis courses to illustrate the method of fixed point iteration for solving equations.

This paper involves a problem confronted by the working diatom taxonomist who must make some basic observations and measurements of the diatom cell wall to achieve an accurate identification. Diatoms are a cosmopolitan group of unicellular algae that are unique in that their cell wall is composed of silica and their food reserve is oil. The cell wall is made of two parts that fit together like a pill box, one part being just slightly smaller than the other. There are two basic configurations: those the symmetry of which is organized about a median line, called Pennate diatoms, and those with symmetry about a point called Centric diatoms. The identification of diatoms to the species level is possible by microscopic observation and measurement of the intricate patterns on the cell wall. Since diatoms form the beginning of many aquatic food chains, they are important to many environmental studies and diatom communities can be used to assess the integrity or health of aquatic ecosystems.

Centric diatoms have uniformly spaced radial markings on the surface which are called striae. A common morphological feature used in identification consists of the number of striae found in a 10 micron arc along the rim of the diatom (equivalently, the striae density). This measurement is made at the margin of the cell by using a reticule in the microscope eyepiece. For large cells the arc and the 10 micron measuring cord become essentially identical. But for small cells, the arc becomes increasingly longer than the 10 micron measuring cord, resulting in a distortion of the measured density of striae. The problem is to determine at what cell size a different method of determining the striae density must be used.



Top. *Stephanodiscus alpinus*. 4000×. Line = 10 microns. Cell 19·5 microns in diameter. The marginal segment is less than 10 per cent longer than the subtending 10 micron cord. Nomarsky optics. Bottom. *Stephanodiscus dubius*. 5100×. Line = 10 microns. Cell 13·0 microns in diameter. The marginal segment is more than 10 per cent larger than the subtending 10 micron cord. Nomarsky optics.

We suppose that the striae density is to be measured by the method above to within a prescribed relative error of ε. If μ is the length of the measuring cord (typically 10 microns) and A is the length of the corresponding arc, then the required condition is:

$$N/\mu - N/A \leqslant \varepsilon N/A$$

where N is the number of observed striae. Setting $a = A/2$ and $c = \mu/2$, this becomes:

$$a \leqslant (1+\varepsilon)c \tag{1}$$

where a and c are related to the radius r of the diatom and the subtended angle $\theta (0 < \theta < \pi/2)$ by:

$$a = r\theta \text{ and } c = r \sin\theta \tag{2}$$

From the last equation we see that for fixed μ (i.e., fixed c), the minimal value of the radius r of a centric diatom, for which the striae density may be measured to a relative accuracy of ε, corresponds to a maximum value of the angle θ. In view of (1) and (2), we are led to find the maximum value of θ in the interval $(0, \pi/2)$ which satisfies:

$$\theta \leqslant (1+\varepsilon)\sin\theta \tag{3}$$

Since

$$\lim_{\theta \to 0+} \frac{\sin\theta}{\theta} = 1 > \frac{1}{1+\varepsilon}$$

the strict inequality is satisfied for all sufficiently small positive θ. On the other hand, under the reasonable assumption that $\varepsilon < \pi/2 - 1$, the inequality is violated at $\theta = \pi/2$. Therefore, by the Intermediate Value Theorem, there is a θ in $(0, \pi/2)$ satisfying:

$$\theta = (1+\varepsilon)\sin\theta \tag{4}$$

Moreover, since $\sin\theta$ is concave down in $(0, \pi/2)$ the solution of (4) is unique and coincides with the maximal θ satisfying (3).

Equation (4) is similar in form to Kepler's equation of astronomy (see for example, [1], p. 47.), the major difference being that the coefficient of $\sin\theta$ in Kepler's equation is the eccentricity of the nearly circular planetary orbit, which is typically of the order of a few hundredths, while the same coefficient in (4) is always greater than one. Kepler used what amounts to the method of iteration to solve his equation which is still used as a standard example of fixed point iteration in texts on numerical analysis (see for example, [2], p. 34.). The method of iteration converges quite rapidly for Kepler's equation since the eccentricity of the orbit, which is much less than one, serves as a contraction constant.

The unique solution of (4) is also a point of attraction for the fixed point iteration

$$\theta_{n+1} = (1+\varepsilon)\sin\theta_n \tag{5}$$

and the convergence of the sequence is monotone. However, the derivative of the iterating function:

$$f(\theta) = (1+\varepsilon)\sin\theta$$

at the solution of (4) is nearly one for small ε which gives rise to a very slowly covergent iterative sequence. This example can therefore be useful in illustrating the desirability of acceleration techniques, such as Aitken's Δ^2-method.

The use of the iteration method (5) gives the following values of θ (to six decimal places) for various choices of the allowable relative error ε:

ε	0·1	0·05	0·01
θ	0·748987	0·538412	0·244097

The radii of the corresponding diatoms may then, in view of (2) and (4), be computed by the formula:

$$r = c/\sin\theta = c(1+\varepsilon)/\theta$$

For the standard length of $\mu = 10$ microns this results in the following radii (displayed to two decimal places):

ε	0·1	0·05	0·01
r	7·34	9·75	20·69

References

[1] Goldstine, H. H., 1977, *A History of Numerical Analysis from the 16th through the 19th Century* (New York: Springer-Verlag).

[2] Conte, S. D., and de Boor, C., 1972, *Elementary Numerical Analysis*, second edition (New York: McGraw-Hill).